Relations

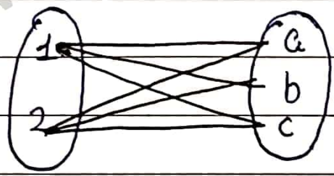
- The relation can be described as a collection of ordered pairs. It is used to relate an object or from one set to the other set, and sets must be non-empty.

- The relation can contain two or more than two sets.

- Relations are derived from cartesian Product.

$$A = \{1, 2\} \quad , \quad B = \{a, b, c\}$$

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$



Let A and B two non empty sets, then a relation R from A to B is a subset of $(A \times B) \rightarrow$ Cartesian Product

$$R \subseteq A \times B$$

Eg.:- $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ → $A \times B$ $= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

$R_1 = \{(1,1), (1,3), (2,1), (2,2)\}$

↳ Relation

$R_{MAX} = A \times B$, $R_{MIN} = \{\phi\}$

$A = m$ $m = 2$

$B = n$ $n = 3$

$A \times B = m \times n$

Relation $= 2^{m \times n}$

$= 2^6$

Total no. of Relations $= 2^{m \times n}$

m = no. of elements of Set A
n = no. of elements of Set B

Binary Relation

A Binary Relation 'R' from A to B is a set of Ordered pairs where first element is from Set A and Second element is from set B.

$A = \{a, b\}$, $B = \{1, 2, 3\}$

$ARB = R = \{(a,1), (a,2), (b,3)\}$

↑
Relation

If the ordered pair of G is reversed, the relation also changes.

Example

$A = \{1, 2\}$, $B = \{3\}$

$A \times B = \{(1,3) \ (2,3)\}$

$R = \{(1,3)\}$

R is binary Relation

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Types of Relations

(1) Reflexive Relation:-

A relation R on a set A is said to be a reflexive if $(a,a) \in R$ for every $a \in A$.

Example:- If $A = \{1,2,3,4\}$

$$R = \{(1,1), (2,2), (1,3), (2,4), (3,3), (3,4), (4,4)\}$$

Reflexive Relation:- $\{(1,1), (2,2), (3,3), (4,4)\} \in R$

(2) Irreflexive Relation:-

A relation R on a set A is said to be irreflexive if $(a,a) \in R$ for every $a \in A$.

Example: $A = \{1,2,3\}$

$$R = \{(1,2), (2,2), (3,1), (1,3)\}$$

The relation R is not irreflexive as $(a,a) \in R$ for some $a \in A$ i.e. $(2,2) \in R$.

* Symmetric ~~Real~~ Relation:-

A Relation R on a set 'A' is said to be Symmetric if xRy then yRx $\forall\ x,y \in A$

i.e. if $(x,y) \in R$ then $(y,x) \in R$ $\forall\ (x,y) \in A$

$A = \{1,2,3,4\}$

$R_1 = \{(1,2), (1,3), (2,1)\ (2,2)\ (3,1)\}$

$R_2 = \{\ \}$

* Anti-Symmetric Relation:-

A Relation 'R' is said to be Anti Symmetric if xRy and yRx then $x=y$ $\forall\ x,y \in A$

i.e. if $(x,y) \in R$ then $(y,x) \in R$, only if $\boxed{x=y}$ $\forall\ x,y \in A$

$A = \{1,2,3\}$

$R = \{(1,2), (2,2), (2,1)\}$

$R_2 = \{(1,1), (2,2), (1,3)\}$ Anti ✓.

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Transitive Relation :-

A Relation R on a set A is said to be transitive if $(x R y)$ and $(y R x)$ then $(x R z)$ $\forall x, y, z \in A$

if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$.

Example :- $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2)\}$ ✓

$R_2 = \{(\underset{x\,y}{1,2})\ (\underset{y\,z}{2,3})\}$ ✗

$R_3 = \{\ \}$ ✓

$R_4 = \{(1,1)\ (1,2)\ (2,1)\}$ ✓

Identity Relation :-

Identity Relation I on set A is reflexive, transitive, and symmetric. So identity relation I is an Equivalence Relation.

Example : $A = \{1, 2, 3\}$
$= \{(1,1), (2,2), (3,3)\}$

Void Relation :- It is given by $R : A \to B$ such that $R = \phi \ (\subseteq A \times B)$.

Void Relation $R = \phi$ is Symmetric and transitive but not Reflexive.

Compatibility Relations

- Compatibility Relation, let R be a relation on a set A. R is a compatibility relation on A if and only if it is $\boxed{\text{reflexive}}$ and $\boxed{\text{Symmetric}}$.

$\boxed{\text{Note}}$

- Compatability Relation is represented by "≈"
- All equivalence relation are compatibility relations.

(i) Reflexive i.e. $(a,a) \in R \quad \forall \; a \in A$
(ii) Symmmetric i.e. $(a,b) \in R$ the $(b,a) \in R, \forall \; a,b \in A$

$\boxed{\text{Example}}$ $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (2,1), (3,2), (1,3), (3,1)\}$

R1 is satisfying all the conditions, so it is Compatible Relation.

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* Equivalence Relation :-

A Relation R on a set A is said to be equivalence if it is reflexive, Symmetric and Transitive.

$A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$ R✓ S✓ T✓

$R_2 = \{(1,1), (2,2), (3,3), (2,1), (1,2)\}$ R✓ S✓ T✓

$R_3 = \{(1,1), (2,2), (3,3), (3,2), (1,3)\}$ R✓ S✗

$R_4 = \{\ \}$

* Partial Order Relation

follows → RAT

A relation 'R' on a set 'A' is said to be partial order if:

(i) R is reflexive i.e. $(a,a) \in A, \forall a \in A$

(ii) R is anti-symmetric i.e. $\forall a, b \in A$ and $(a,b) \in R \wedge (b,a) \in R$, then $a = b$

(iii) R is transitive i.e., $\forall a, b, c \in A$ and $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$

Example $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$ R✓ A✓ T✓

$R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ A✗

$R_3 = \{(1,1), (1,2), (2,3), (1,3)\}$ ✗

$R_4 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ R✓ A✓ T✓

So, R_1 and R_4 are Partial Order Relation.